Solving Distributed CSPs using Dynamic, Partial Centralization without Explicit Constraint Passing

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ABSTRACT

Dynamic, partial centralization has received a considerable amount of attention in the distributed problem solving community. As the name implies, this technique works by dynamically identifying portions of a shared problem to centralize in order to speed the problem solving process. Currently, a number of algorithms have been created which employ this simple, yet powerful technique to solve problems such as distributed constraint satisfaction (DCSP), distributed constraint optimization (DCOP), and distributed resource allocation.

In fact, one such algorithm, Asynchronous Partial Overlay (APO), was shown to outperform the Asynchronous Weak Commitment (AWC) protocol, which is one of the best known methods for solving DCSPs. One of the key differences between these two algorithms is that APO, as part of the centralization process, uses explicit constraint passing. AWC, on the other hand, passed *nogoods* because it tries to provide security and privacy. Because of these differences in underlying assumptions, a number of researchers have criticized the comparison between these two protocols.

This paper attempts to resolve this disparity by introducing a new AWC/APO algorithm called Nogood-APO that like AWC uses nogood passing to provide security and privacy and like APO uses dynamic partial centralization to speed the problem solving process. Like its parent algorithms, this new protocol is sound and complete and performs nearly as well as APO, while still outperforming AWC, on distributed 3-coloring problems. In addition, this paper shows that Nogood-APO provides more privacy to the agents than both APO and AWC on all but the sparsest problems. These findings demonstrate that a dynamic, partial centralization-based protocol can provide privacy and that even when operating with the same assumptions as AWC still solves problems in fewer cycles using less computation and communication.

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Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

General Terms

Algorithms, Design, Performance

Keywords

Distributed Constraint Satisfaction, Cooperative Mediation, Dynamic Partial Centralization

1. INTRODUCTION

Over the years, distributed problem solving has received a great deal of attention for a number of reasons. The most compelling reasons are that some problems are naturally distributed, multiple processor can compute solution faster, and privacy and security can be maintained. These reasons can often be quite contradictory because, for example, the more information an agent is willing to reveal upfront as part of the problem solving process, the faster a solution can be computed.

One methodology for solving distributed problems, called dynamic, partial centralization tries to solve naturally distributed problems in the fastest manner possible by performing focused, incremental and asynchronous centralization of portions of a shared problem. Several protocols have already been created that use this hybrid centralized/distributed search technique and have been shown to outperform existing protocols on a large number of distributed problems.

One of the key characteristics of each of these algorithms, however, is that the agents have to be willing to directly reveal a great deal of information to each other. For example, in Asynchronous Partial Overlay (APO) [8], agents willingly reveal their variable's constraints and domain whenever requested. The Asynchronous Weak Commitment (AWC) protocol [15], on the other hand, only reveals information about a variable's constraints and domain, using a *nogood*, when it reaches a *deadend* in the problem solving process. The willingness to reveal information is one reason, although not the only one, that APO outperforms AWC across a wide spectrum of problem sizes and difficulties.

The purpose of this paper is to present a new hybrid AWC/APO algorithm, called Nogood-APO. Like APO, this new algorithm uses dynamic, partial centralization and like AWC only reveals information, in the form of a nogood, when necessary during the problem solving process. The two main goals in creating this algorithm are to show that

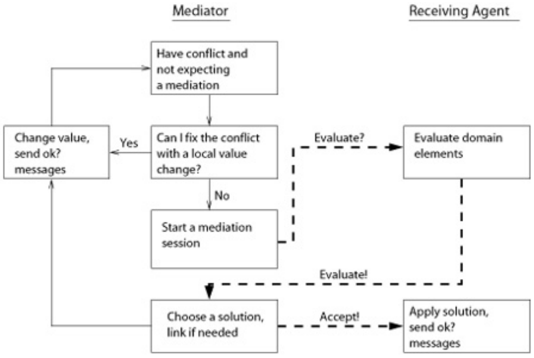


Figure 1: The basic APO protocol.

although partial centralization involves revealing knowledge in order to solve a shared problem, the knowledge that is exposed can be minimized, obscured, or revealed in an incremental manner. The second goal is to demonstrate that even when constraints are not explicitly revealed, that dynamic, partial centralization still outperforms the AWC-like trial-and-error approach to solving distributed problems.

The rest of this paper is organized as follows. In the next section, we will introduce the distributed constraint satisfaction problem. We will go on to describe the Nogood-APO algorithm, give an example of its execution on a simple problem, and mention the issues of soundness and completeness. We will then present the setup for our experimental evaluation followed by the results. Finally, we will present our conclusions and future work.

2. DISTRIBUTED CONSTRAINT SATISFACTION

A Distributed Constraint Satisfaction Problem (DCSP), $P = \langle V, A, D, R \rangle$, consists of the following [16]:

- A set of n variables $V = \{x_1, \ldots, x_n\}$.
- A set of g agents $A = \{a_1, \ldots, a_g\}$
- discrete, finite domains for each of the variables $D = \{D_1, \ldots, D_n\}$.
- a set of constraints $R = \{R_1, \ldots, R_m\}$ where each $R_i(d_{i1}, \ldots, d_{ij})$ is a predicate on the Cartesian product $D_{i1} \times \cdots \times D_{ij}$ that returns true iff the value assignments of the variables satisfies the constraint.

The problem is to find an assignment $S = \{d_1, \ldots, d_n | d_i \in D_i\}$ such that each of the constraints in R is satisfied. DCSP, like its centralized counterpart CSP, has been shown to be NP-complete, making some form of search a necessity.

In this work, we focus on the case where each agent is assigned a single variable and the constraints are binary. Since each agent is assigned a single variable, we will refer to the agent by the name of the variable it manages. Because the constraints are binary, we can refer to the graph created by representing variables as vertices and constraints as edges as the *constraint graph.* In addition, two variables are considered to be *neighbors* if they share a constraint.

3. THE PROTOCOLS

3.1 Asynchronous Weak Commitment (AWC)

The Asynchronous Weak Commitment (AWC) protocol is heavily based on its predecessor the Asynchronous Backtracking (ABT) protocol [17]. ABT works by assigning each agent a priority value. These priority values establish an absolute ordering amongst the agents that is used to control the search process. Agents perform the search by sending *value* messages to lower priority agents they are linked with. Value messages inform these lower priority agents about the variable values of higher priority agents. The agents use these values to determine if any of their domain values can satisfy their constraints with higher priority agents. Whenever the values of the higher priority prevent them from assigning their variable a conflict free value, the agent generates a *nogood* message.

Nogoods are composed of a set variable/value pairs that indicate that the combination of the variable assignments cannot be part of a satisfying solution. A nogood can be thought of as an implied constraint. After generating a nogood, it is sent to all the agent that are contained within it. Upon receiving a nogood, agents perform a linking step with any agent that is listed in the nogood and was previously unknown. This step is necessary to ensure the completeness of the search. Initially, the linking structure mirrors the constraint graph, but because of linking as a result of nogoods, can quickly grow causing higher priority agents to send value messages to a large number of agents.

Like centralized backtracking algorithms, the ordering of the agents (variables) in ABT strongly affects the speed of the search. To overcome this problem, Yokoo created the AWC protocol [15]. The AWC algorithm is a variant of the ABT algorithm that allows the agents to re-prioritize themselves using the weak-commitment search heuristic [14]. This heuristic strategy basically says that whenever a backtrack occurs, that variable that triggered the backtrack should be moved up in the search tree. The principle idea behind this technique is to identify variables that are at the center of complex or critical constraints and assign them values first. Their values can then act as constraints on less critical variables instead of the other way around. In practice, this techniques has been shown to be quite effective in reducing the overall runtime of DCSP searches.

A later addition to the AWC protocol was the use of resolvent-based nogood learning [6]. This technique works by selecting, for each of the variable's possible values, one nogood that prohibits that value. These nogood are then merged together to form a new nogood. If the nogoods are selected wisely, they can actually generate smaller, more powerful nogoods.

3.2 Asynchronous Partial Overlay (APO)

Conceptually, APO is based on the *cooperative mediation* paradigm [8]. Cooperative mediation entails three main principles. The first is that agents use local, centralized search to solve portions of the overall problem. Second, agents should use experience to dynamically increase their understanding of their role in the overall problem. Third, agents should overlap the knowledge that they have to promote coherence. Together these three ideas create a powerful paradigm which has been applied to several distributed problems [10, 11].

The basic APO algorithm is presented in Figure 1. The APO algorithm works by constructing two main data structures; the *good_list* and the *agent_view*. The *agent_view* holds the names, values, domains, and constraints of variables to which an agent is linked. The *good_list* holds the names of the variables that are known to be connected to the owner by a path in the constraint graph.

As the problem solving unfolds, the agents try to solve the subproblem they have centralized within their *good_list* or determine that this subproblem is unsolvable (indicating that the entire problem is overconstrained). To do this, whenever an agent recognizes a constraint violation involving its variable, it takes the role of the mediator and attempts to change the values of the variables within the mediation session to achieve a satisfied subsystem. When this cannot be achieved without causing a violation for agents outside of the session, the mediator links with those agents assuming that they are somehow related to the mediator's variable. This step increases the size of the *good_list*. This process continues until one of the agents finds an unsatisfiable subsystem, or all of the conflicts have been removed.

Like AWC, agents that use APO have a dynamic priority value that is used to determine which agent mediates when a conflict is detected. Currently, the heuristic for setting this priority value is to use the size of the subproblem that the agent knows. Although one could conceive of a number of other heuristics which optimize different metrics, this particular heuristic was chosen to minimize the number of parallel cycles needed to compute a solution. Benisch and Sadeh, for instance, developed an inverted mediator selection strategy that improves the parallelism of the protocol at the expense of requiring additional communication cycles [1].

When an agent links in APO, the agents exchange the domain values, D_i, and constraints, $\forall R_i \; x_i \in R_i$, on the their variable. In many environments, particular in ones where every agent is trusted and cooperative, the open exchange of this knowledge is quite acceptable and leads to significant improvements in the runtime of the algorithm. However, there are times when directly exchanging this information is impossible due to privacy or security.

3.3 Nogood-APO

The Nogood-APO (NAPO) algorithm is very similar in nature to the APO algorithm. The key difference is that instead of directly exchanging constraints, the agents exchange *nogoods* as part of the problem solving process.

By exchanging nogoods, the agents gain two things. First, because the agents incrementally reveal information, they may not have to reveal all of the details about their constraints in order to solve a problem. This is particular important in domains where the variables have very large domains. The second is that agents can obscure their constraints by padding the most minimal nogood with additional variable/value pairs. By padding them in this way, it is harder for another agent to actually know the details of the constraints, but it slows the execution of the algorithm because it is harder to identify when the problem is unsolvable.

There are several secondary effects of changing the algorithm in this way. The most important is that the agents need to maintain a *nogood list*. Like AWC, the size of the nogood list can grow quite large (exponential in the worst case), especially if agents try to hide their direct constraints by padding their nogoods. However, if the agents are willing to exchange nogoods that are directly derived from their constraints, the size of the nogood list becomes quite manageable being directly related to the number and complexity of the constraints as opposed to the number of possible assignments to the variables.

Initialization.

Like APO, on startup, the agents are provided with the value (they pick it randomly if one isn't assigned) and the constraints on their variable. Using these constraints, the agents derived their direct nogoods and place them in their nogood lists. Unlike APO however, initialization proceeds by having each of the agents send out an "ok?" message to its neighbors. The content of this message is considerably different from the "ok?" messages in APO. In NAPO, the agents send their current priority, the value of their variable, their variable's current domain, and the current set of violated nogoods from their nogood list that involve their variable.

Agents send their domain values as part of the "ok?" message because it ensures that the mediator always has the current set of allowable values for the variables in its good_list. This is particularly important if an agent calculates that one of its values is not arc-consistent. This can be thought of as the agent deriving a unary nogood which disallows one of its variable's values.

The "ok?" message also includes the set of currently violated nogoods that include the agent's variable. There are two reasons for including this information. First, when this set is empty, it indicates that the agent does not wish to mediate. Second, as will be illustrated later, this information is used to ensure that mediators are informed of inadvertent nogood violations that result from changing the values of multiple variables in a session without knowing that they are related to one another.

When an agent receives an "ok?" message (either during the initialization, through a later link request, or as a state update), it records the information in its agent_view and adds the variable to the good_list if it can. A variable is only added to the good_list if it shares a nogood with a variable that is already in the list. This restriction ensures that the graph created by the variables in the good_list always remains connected.

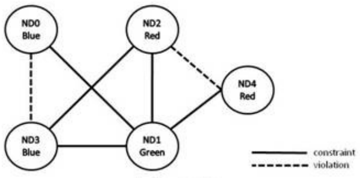
Checking the agent view.

Whenever the agent receives a message that indicates a possible change to the status of its variable, it checks the current agent_view (which contains the assigned, known variable values) to identify violated nogoods. If, during this check, an agent finds a violation and has not been told by a higher priority agent that they want to mediate, it assumes the role of the mediator.

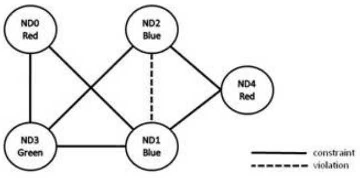
As the mediator, an agent first attempts to rectify the violation(s) by changing its own variable. This simple, but effective technique prevents sessions from occurring unnecessarily, which stabilizes the system and saves messages and time. If the mediator finds a value that removes the violations, it makes the change and sends out an "ok?" message to the agents in its agent_view. If it cannot find a nonconflicting value (it's at a deadend), it starts a mediation session.

Mediation.

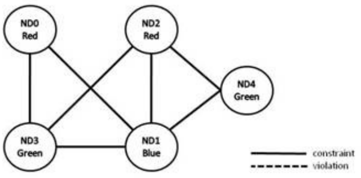
The most complex and certainly most interesting part of the protocol is the mediation. The mediation starts with the mediator sending out "evaluate?" messages to each of the



(a) Initial.



(b) After first mediation.



(c) After second mediation.

Figure 2: Example 3-coloring problem with 5 variables and 7 not-equals constraints.

agents in its good_list. The purpose of this message is twofold. First, it informs the receiving agent that a mediation is about to begin and tries to obtain a lock from that agent. This lock prevents the agent from engaging in two sessions simultaneously or from doing a local value change during the course of a session. The second purpose of the message is to obtain information from the agent about the effects of making them change their local value. This is a key point.

When an agent receives a mediation request, it will respond with either a "wait!" or "evaluate!" message. The "wait" message indicates to the requester that the agent is currently involved in a session with a higher priority agent or is expecting a request from an higher priority agent. If the agent is available, it labels each of its domain elements with the nogoods that would be violated if it were asked to take that value which is returned in an "evaluate!" message.

When the mediator has received either a "wait!" or "evaluate!" message from all of the agents that it has sent a request to, it computes a solution using a Branch and Bound search [3]. The goal of the search is to find a conflict-free solution for the variables in the session and to minimize the number of conflicts for variables outside the session (like the min-conflict heuristic [13]). During this search, new nogoods can be derived using nogood learning [4]. These nogoods are recorded in the nogood list and can be used during subsequent searches to prune the search space.

If no satisfying assignments are found, the agent announces that the problem is unsatisfiable and the algorithm terminates. If a solution is found, "accept!" messages are sent to the agents in the session and "ok?" messages are sent to the agents that are in its agent_view, but, for whatever reason, were not in the session, and to any agent that is not in its agent_view, but it caused conflict for as a result of selecting its solution.

3.4 Example Execution

Consider the 3-coloring problem in Figure 2a. In this problem there are 5 variables, each assigned to an agent and 7 constraints which represent the "not equals" predicate. Being a 3-coloring problem, the variables can only take the value red, green, or blue. There are currently two constraint violations, between ND2 and ND4 and between ND0 and ND3.

On initialization, each of the agents adds nogoods to their nogood lists for the constraints that they have on their variable. They then send "ok?" messages to the agents with whom they share constraints (their neighbors).

Once the initialization has completed, each of the agents checks its *agent_view* to determine if its variable is involved in a violation. In this case, ND0, ND2, ND3, and ND4 determine that have a conflict. Because of the priority ordering, ND4 (priority 3) waits for ND2 (priority 4) to mediate. ND0 (priority 3) and ND2 wait for ND3 (priority 3 tie broken by name). ND3, knowing it is higher priority than ND0 and ND2, first checks to see if it can resolve its conflicts by changing its value, which it can't. It then starts a mediation session and sends "evaluate?" messages to ND0, ND1, and ND2.

Upon receiving the "evaluate?" messages, ND0, ND1, and ND2 evaluate their domain elements to identify the nogoods that would be violated by each of them. This information is then returned to ND3 in an "evaluate!" message. The following are the labeled domains for the agents in the session with ND3:

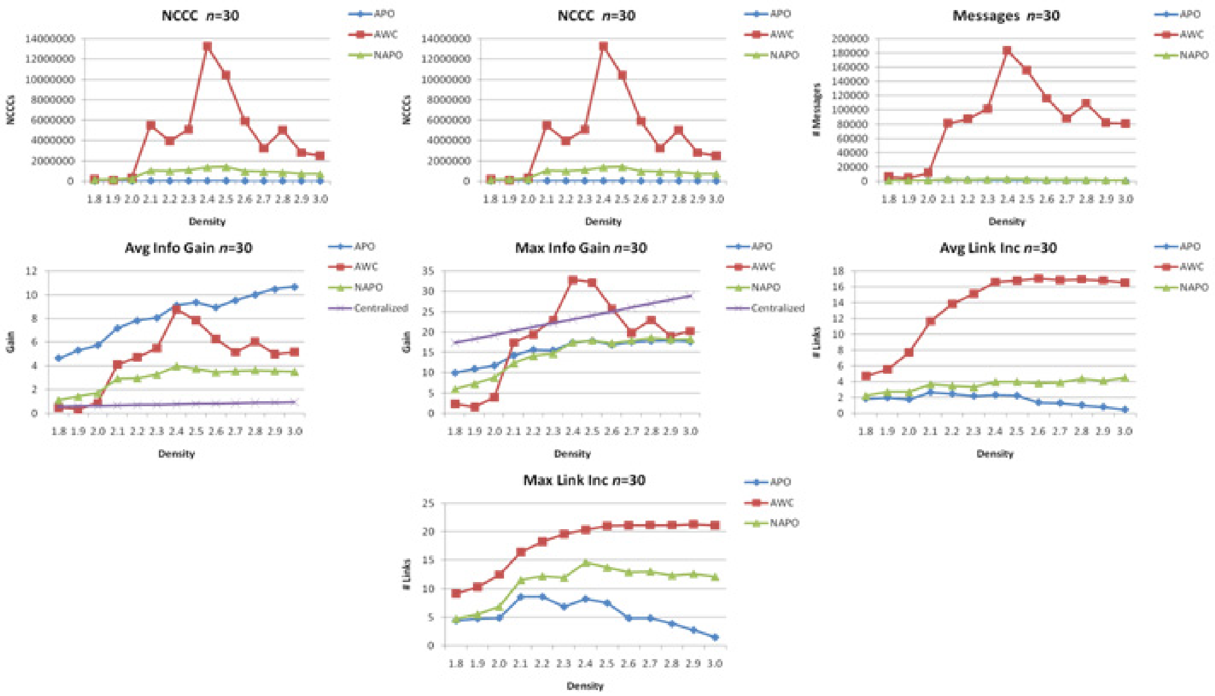
- ND0
  Green violates (ND0=G,ND1=G)
  Blue violates (ND0=B,ND3=B)
  Red causes no violations
- ND1
  Green cause no violations
  Blue violates (ND1=B,ND0=B) and (ND1=B,ND3=B)
  Red violates (ND1=R,ND2=R) and (ND1=R,ND4=R)
- ND2
  Green violates (ND2=G,ND1=G)
  Blue violates (ND2=B,ND3=B)
  Red violates (ND2=R,ND4=R)

ND3 computes a solution that changes the values of all of the variables in the session (see Figure 2b). Based on the information that ND3 obtained from the "evaluate!" messages, it believes that this solution solves its subproblem and causes no conflicts for agents outside of the session. ND3 sends "accept!" message to the agents in the session.

After receiving the "accept" messages, each agent changes its value and checks its agent_view. This time, ND1 and ND2 are in conflict. This happened because ND3 changing their values to blue, inadvertently causing the violation. To prevent this from happening again, the "ok?" messages that are sent by ND1 and ND2 include their current conflict set. This allows ND3 to learn of the relationship between ND2 and ND1 so it doesn't repeat the same error.

ND1, the higher priority (priority 5) agent, cannot solve the conflict by making a local value change, so it starts a mediation session. Below are the responses to the "evaluate?" messages sent by ND1:

- ND0
  Green violates (ND0=G,ND3=G)
  Blue violates (ND0=B,ND1=B)
  Red causes no violations


NCCC n=30

NCCC n=30

Messages n=30

Avg Info Gain n=30

Max Info Gain n=30

Avg Link Inc n=30

Max Link Inc n=30

Figure 3: Phase transition results for 30 node graphs of various density.

- ND2
  Green violates (ND2=G,ND3=G)
  Blue violates (ND2=B,ND1=B)
  Red violates (ND2=R,ND4=R)
- ND3
  Green causes no violations
  Blue violates (ND3=B,ND1=B)and (ND3=B,ND2=B);
  Red violates (ND3=R,ND0=R)
- ND4
  Green causes no violations
  Blue violates (ND4=B,ND2=B)and (ND4=B,ND1=B)
  Red causes no violations

ND1 computes a solution which changes its value to green and ND2's to red and sends "accept!" messages. All of the agent's check their agent_view and find no conflicts so the problem is solved (Figure 2c).

3.5 Soundness and Completeness

The soundness and completeness of the NAPO algorithm are derived directly from the soundness and completeness of APO. We refer the reader to [9] and [5] for the complete details of the proofs for APO. Here is a basic outline of the proof for NAPO:

- If at anytime an agent identifies a constraint subgraph that is not satisfiable, it announces that the problem cannot be solved. Half of the soundness.
- If a nogood is violated, someone will try to fix it. The protocol is dead-lock and live-lock free. The other half of the soundness proof.
- Eventually, in the worst case, one or more of the agents will centralize the entire problem and will derive a solution, or report that no solution exists. This is done by collecting nogoods from both "evaluate!" messages and "ok?" messages. This ensures completeness.

4. EMPIRICAL EVALUATION

4.1 Experimental Setup

To test the NAPO algorithm, we implemented the AWC, APO, and NAPO algorithms and conducted experiments in the distributed 3-coloring domain. The particular AWC algorithm we implemented can be found in [18] which includes the resolvent *nogood* learning mechanism described in [6]. We conducted two sets of experiments.

In the first set of experiments we compared the algorithms using 30 variable, randomly generated graph coloring problems while varying the edge densities across the known phase transition for 3-coloring problems [2]. In the second set of experiments, we tested the scalability of the algorithms by varying the size of the problems from 15 to 60 variables in the three major regions of the phase transition. Each data point represents an average over 30 randomly generated problems. Each algorithm was given the same problems with the same initial variable assignments to minimize variance. The algorithms were allowed to run for up to 1,000 cycles. This upper limit only affected the AWC protocol, which frequently could not finish on larger, higher density problems. A total of 2,250 test runs were conducted.

During these tests we measured the number of messages, cycles, and non-concurrent constraint checks (NCCCs) [12] used by the algorithms. During a cycle, incoming messages are delivered, the agent is allowed to process the information, and any messages that were created during the processing are added to the outgoing queue to be delivered at the beginning of the next cycle. The actual execution time given to one agent during a cycle varies according to the amount of work needed to process all of the incoming messages. We also instrumented the algorithms to measure the number of non-concurrent constraint checks used during each cycle.

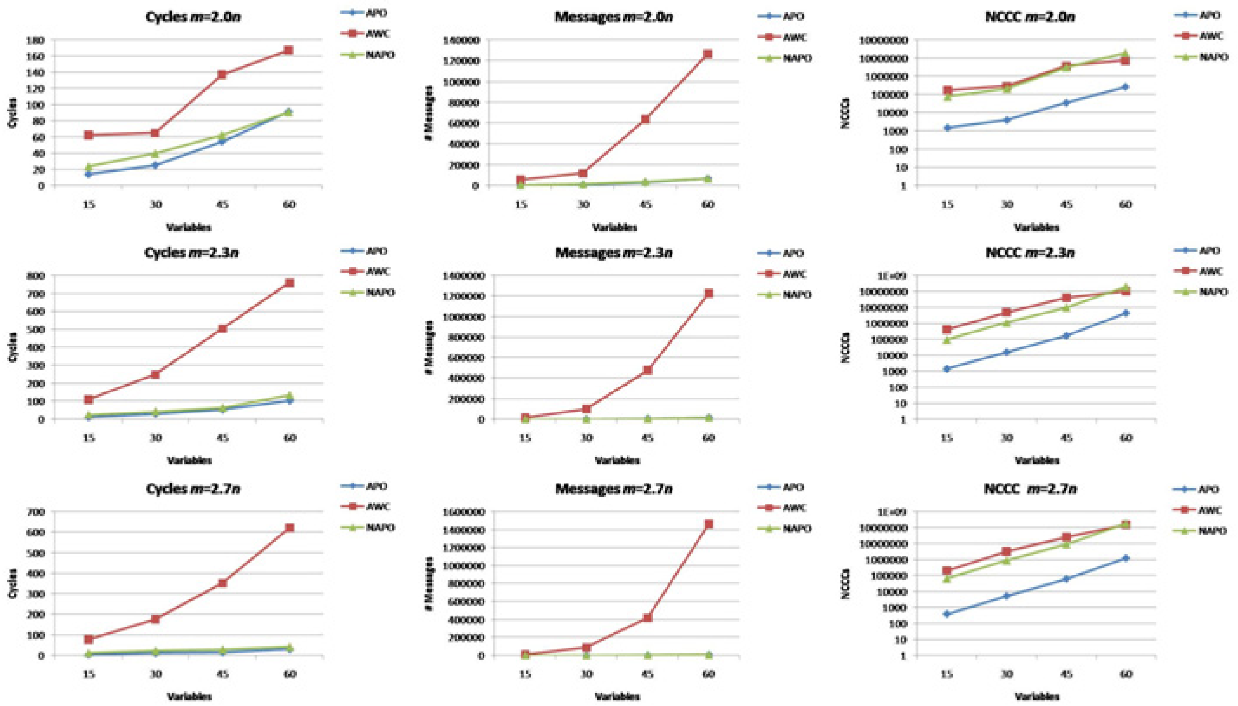


Figure 4: Scalability cost results for AWC, APO, and NAPO.

This measure has gained popularity in the DCSP community because it provides an implementation independent view of the parallel computation usage of a protocol.

In addition to these standard measures of computation and communication cost, we also gathered data to quantify the information that the agents revealed to one another during the problem solving process. One measure we used was to count the number of links that the protocols created during execution. This metric provides insight into "who" the agents send information to in order to solve the problem. We also wanted to measure "what" and how much information was being sent. To do this we used the following measure of information gain:

$$gain(a_i) = \sum_{ng \in nogoods-rcvd_i} \frac{1}{|D_i|^{|ng|}} \quad (1)$$

where a_i is an agent, $nogoods - rcvd_i$ is the set of unique nogoods that have been received from other agents by a_i, $|D_i|$, is the size of the domain, and $|ng|$ is the size of an individual nogood based on the number of variable/value pairs it contains. The logic behind this equation is that the power of a nogood can be measured based on the number of potential solutions that it invalidates in the search space. Shorter nogoods are more powerful because they are more general and eliminate a larger number of value combinations. This metric is similar to the Value of Possible States (VPS) metric developed by Meheswaran et al. [7]. For both of these metrics we determined the average across the agents, measuring the distribution of information gain, as well as the maximum value for any single agent, measuring the amount of centralization.

To provide a frame of reference, we also included data for the average and maximum information gain had the agents elected a leader and centralized the problem. The centralized maximum and average information gain can easily be computed as:

$$max\_gain(a) = \frac{m(n-1)}{n * |D_i|} \quad (2)$$

$$avg\_gain(a) = \frac{max\_gain(a)}{n} \quad (3)$$

4.2 Results

The result of the phase transition experiments can be seen in Figure 3. These graphs show that AWC outperforms both APO and NAPO on very sparse problems, but on problems at or above the phase transition, the story is quite different. APO uses the least number of NCCCs, cycles, and messages with NAPO using slightly more. These results seem to contradict the findings of presented by Grinshpoun and Meisels [5] who reported that APO used more NCCCs on medium density problems across various levels of constraint tightness. The discrepancy between these results can likely be explained by the difference in the experiments that were conducted. Grinshpoun and Meisels used general CSP instances where the variables have large domains ($|D_i| = 10$) as opposed to the small domain of the variables ($|D_i| = 3$) and fixed tightness of the constraints ($p_2 = 0.33$) in 3-coloring. The large domains create equally large branching factors that severely impact the branch and bound solver used at the core of APO.

When looking at the results for information exchange, the nature of the protocols becomes apparent. APO, which uses explicit constraint passing, has the worse average information exchange across the entire transition, centralizes about 50% of the problem within a single agent, but creates the least number of new links. NAPO has the lowest average information gain, is equivalent to APO in the amount of in-

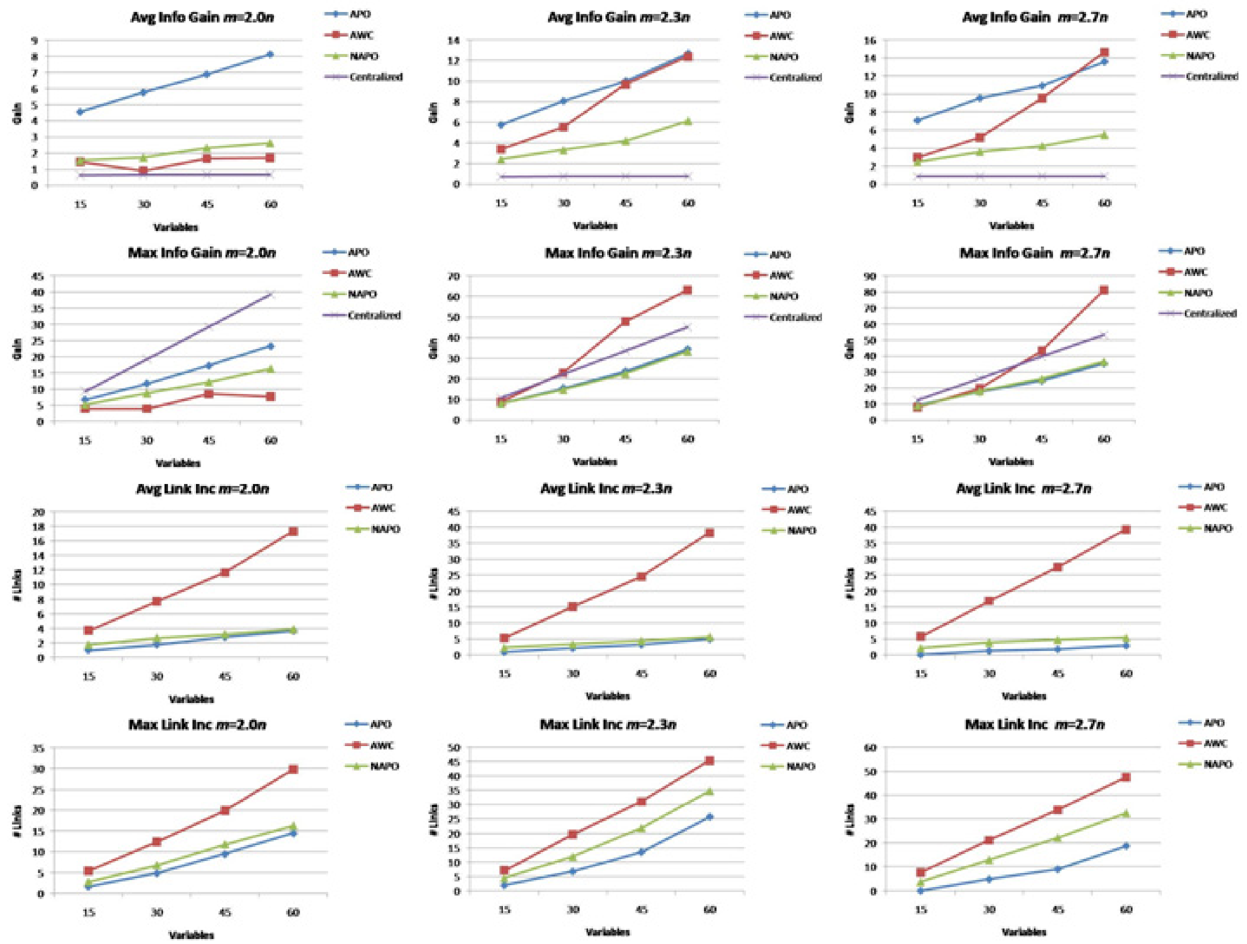


Figure 5: Scalability information results for AWC, APO, and NAPO.

formation centralized in a single agent, and produces more links than APO. This can be interpreted as meaning that NAPO centralizes as much as APO, but does it in a more intelligent manner. AWC has the least average information gain on very sparse problems, but within the phase transition performs worse than NAPO and actually approaches APO. AWC has very minimal centralization on sparse problems, but as the density increases, the agent with the maximum information gain actually gets more information than if the problem had just been completely centralized. At first, this doesn't appear to make sense, but AWC agents not only send original constraints, they also send implied constraints. So the agent with the maximum information gain is not only being told the other agents' constraints, it is being told about constraints that are learned by the other agents as well. AWC also creates more links meaning that agents are exchanging information with more of their peers than APO and NAPO

The results of the scalability experiments can be seen in Figures 4 and 5. The results for the cost metrics are as expected with APO using the least cycles, communication, and NCCCs of the three protocols. The protocol cost for NAPO is somewhere between APO and AWC. In the NCCCs category it appears that AWC and NAPO are competitive. However, one should keep in mind that many of the AWC runs did not actually complete on the 60 node test cases because they did not find a solution within 1,000 cycles. So the results for AWC in these graphs are skewed toward being lower than they actually are.

The results for scalability of information gain also present some interesting findings. They show that on sparse problem, both AWC and NAPO have less average information gain than APO. However, on denser problems, AWC becomes less scalable having a rapid increase in average information gain that exceeds even APO. The same trend holds true when looking at maximum information gain. AWC is dominate on sparse problems, but on dense examples has poor scalability. APO performs best overall in the number of new links it creates, with NAPO in the middle and AWC creating the most links.

The take-home message from these experiments are not directly straightforward, but can be summarized as follows:

- On sparse 3-coloring problems, the AWC protocol exchanges the least amount of information in order to compute a solution, but takes more cycles, uses more messages, creates more links, and performs more NCCCs than APO.
- On dense 3-coloring problems, AWC exchanges more information, to more agents, uses more cycles, more messages, and more NCCCs than either NAPO or APO.
- If you are solely concerned about speed then APO is

your best choice.

- If you are willing to trade speed for privacy than NAPO is the best choice on everything except very sparse problems.
- The speedups associated with partial centralization cannot be directly attributed to explicit constraint passing alone. Even when nogoods are exchanged, the algorithm performs as well or better than the distributed backtracking-based search.

5. CONCLUSIONS AND FUTURE WORK

In this paper, we presented a new hybrid AWC/APO algorithm called Nogood-APO. As was shown in experimentation, this algorithm, like APO, outperform AWC on all but the simplest 3-coloring problems across various size and density on several metrics. By creating this algorithm, we showed that constraint passing is not necessary in an algorithm that is based on dynamic, partial centralization and that the likely reason why algorithms like APO outperform AWC is the combination of distributed/centralized search techniques they use.

A number of questions are raised as a result of this work. First, and foremost, it revives the competition between DCSP algorithms that are based on partial centralization and distributed backtracking because for the first time, we have examples that are designed using the same basic assumptions. It also identifies another dimension for doing scalability experiments, namely the size of the variable's domains. As these results indicate, on problems that have variables with small domains, the performance characteristics AWC and APO are quite different then they are on domains with larger domains. This may point to areas for improvement in both of these protocols.

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